Resonant Amplification of Gauge Fields in Expanding Universe

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We investigate the possibility that gauge fields are amplified in an expanding universe by parametric resonance, during the oscillatory regime of a scalar field to which they are coupled. We investigate the coupling of gauge fields to a charged scalar field and to an axion. For both couplings, gauge fields fluctuations undergo exponential instabilities. We discuss how the presence of other charges or currents may counteract the resonance, but we argue that in some cases the resonance will persist and that hence this mechanism could have some relevance for the problem of large scale primordial magnetic fields.

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I. INTRODUCTION

In these last years the concept of parametric resonance (henceforth PR) has played a fundamental role in cosmology for the development of the theory of preheating [1], the explosive decay of the inflaton after inflation. There have been investigations of several types of fields coupled to the inflaton: scalars [1], fermions [2], gravitational waves [3], scalar gravitational perturbations [4], gravitinos [5].

Systems which includes *nearly* [6] conformal invariant fields can be efficiently amplified in an expanding universe [7]. This occurs because the Robertson-Walker metric is conformally related to the flat metric, and therefore the equations of motion can be mapped into similar ones in Minkowski space-time. In these systems the resonance can be efficient without resorting to large couplings [8] (it is sufficient to have $q \sim \mathcal{O}(1)$, where q is the amplitude of the driving term). The resonance is not weakened by the expansion of the universe and it persists until nonlinear effects will shut off it [7].

It is interesting to investigate the effect of PR on fields which are conformally coupled to gravity. For these fields particle production due to the expansion of the universe is absent [9]. Therefore PR can be an alternative and efficient way to amplify fluctuations of these fields. Examples of conformally invariant fields are massless vector fields and massless fermions [9]. In the case of fermion the Pauli blocking is present, but this does not prevent the occurring of novel interesting effects [2]. For a massless vector field an exponential Bose enhancement occurs. In this Letter we investigate the evolution of gauge field coupled to a charged scalar field and to an axion.

II. SCALAR ELECTRODYNAMICS

The Lagrangian density for scalar electrodynamics is given by

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} - (D_\mu\Phi)^*(D^\mu\Phi) - V(\Phi^*\Phi) \qquad (1)$$

where $D_\mu = \nabla_\mu - ieA_\mu$ is the gauge and metric covariant derivative, A_μ is the gauge potential and $F_{\mu\nu}$ is the antysimmetric field tensor, defined as $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. From this Lagrangian the equations of motion are:

$$-\Box\Phi + 2ieA_\mu\partial^\mu\Phi + ie\Phi\nabla_\mu A^\mu + \frac{\partial V}{\partial\Phi^*} + e^2 A_\mu A^\mu \Phi = 0 \qquad (2)$$

$$\nabla_\nu F^{\nu\mu} = -4\pi j^\mu + 8\pi e^2 A^\mu |\Phi|^2 \qquad (3)$$

where $j_\mu = ie(\Phi\partial_\mu\Phi^* - \Phi^*\partial_\mu\Phi)$. We shall consider inhomogeneous linearized fluctuations around homogeneous quantities $f(t,\mathbf{x}) = \bar{f}(t) + \delta f(t,\mathbf{x})$ (where f is any field) in the Robertson-Walker metric $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$. We shall study the evolution of the gauge field in the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$. This gauge singles simply out the dynamical degrees of freedom for the problem (1) and its close relation to the gauge invariant formalism has been pointed out in [10]. By writing the homogeneous condensate as $\bar{\Phi}(t) = e^{i\Theta(t)}\rho(t)/\sqrt{2}$ one has at the homogeneous level:

$$\ddot{\rho} + 3H\dot{\rho} + \frac{\partial V}{\partial\rho} + e^2\frac{|\bar{\mathbf{A}}|^2}{a^2}\rho = 0 \qquad (4)$$

plus an equation of motion for $\bar{\mathbf{A}}(t)$ and the constraint $\bar{A}_0 = \dot{\Theta}/e$. We shall consider fluctuations $\delta A_\mu(t,\mathbf{x})$ around a background $\bar{A}(t)$ that is chosen to be zero:

$$\delta\ddot{\mathbf{A}}_T + H\delta\dot{\mathbf{A}}_T - \frac{\nabla^2}{a^2}\delta\mathbf{A}_T + 4\pi e^2\rho^2\delta\mathbf{A}_T = 4\pi\delta\mathbf{j}_T \qquad (5)$$

$$-\frac{\nabla^2}{a^2}\delta A_0 = 4\pi\delta j_0 - 4\pi e^2\rho^2\delta A_0 \,, \qquad (6)$$

where T denotes the transverse part of a vector and $\delta\mathbf{j}_T$ is a source term which can be different from zero only if one consider quantum or statistical correlation of the currents [11] because of the symmetry of the space-time background. The constraint equation (6) in Fourier space is

$$\left[\frac{k^2}{a^2} + 4\pi e^2 \rho^2\right] \delta A_{0\,k} = 4\pi \delta j_{0\,k} \tag{7}$$

with $\delta j_{0\,k} = e(\rho\delta\dot{\phi}_{2\,k} - \dot{\rho}\delta\phi_{2\,k})$ where $\delta\phi = (\delta\phi_1 + i\delta\phi_2)/\sqrt{2}$.

We consider now the homogeneous part of the differential equation (5) in Fourier space:

$$\delta \mathbf{A}''_{Tk} + \omega^2_{Tk} \delta \mathbf{A}_{Tk} = 0 \tag{8}$$

where $\omega^2_{Tk} = k^2 + 4\pi e^2 a^2 \rho^2$ and the prime denotes the derivative with respect the conformal time η ($d\eta = dt/a$). The general solution y of the inhomogeneous differential equation (5) can be obtained through the Green function method [12]:

$$\mathbf{y} = \mathbf{y}_h + 4\pi e^2 \int^{\eta} \left[\frac{y_1(\eta')y_2(\eta) - y_1(\eta)y_2(\eta')}{W}\right] a^2 \delta\tilde{\mathbf{j}}_{Tk} d\eta\prime \tag{9}$$

where y_h is the homogeneous solution, y_1, y_2 are the two linear independent homogeneous solutions and W is their wronskian.

Equation (8) describes a harmonic oscillator with time dependent frequency: during the oscillation of the complex scalar field Eq. (8) can be reduced to a Mathieu-like equation [13]. The solutions to this type of equation show an exponential instability $\propto e^{\mu_k \eta}$, for some interval of frequencies, called *resonance bands*. There is a known correspondence at the level of equations of motion between Eq. (8) and the equation of a massless real scalar field χ (rescaled by its conformal weigth) interacting with a real scalar field ϕ by a term $g^2\phi^2\chi^2$:

$$(a\chi)''_k + \left[k^2 + g^2 a^2 \phi^2 + (\xi - \frac{1}{6}) a^2 R\right] (a\chi)_k = 0\,, \tag{10}$$

where $R = 6a''/a^3$ is the Ricci curvature. In the case of conformal coupling ($\xi = 1/6$) this correspondence is exact, while for a quartic potential for ϕ (or Φ) this correspondence is *nearly* exact since $R \simeq 0$ [7]. The behaviour of parametric resonance depends strongly on the time behaviour of the homogeneous scalar field, which on turn depends on the form of the potential $V(\Phi)$. In the following we consider power law potentials as:

$$V = \lambda_n (\Phi^* \Phi)^{2n} \tag{11}$$

and we briefly illustrate the different behaviours depending on the parameters n and λ_n, restricting ourselves to the case in which the scalar field is the dominant component of energy density in the universe [14].

For a quadratic potential ($n = 1$) the driving term $a^2\rho^2$ in Eq. (8) decays as η^{-2}. PR is efficient for $4\pi e^2 \rho^2 \gg \lambda_1$: the resonance is stochastic and broad, and the largest Floquet exponent μ_k occurs for small k fluctuations [8]. In the conformally invariant quartic case ($n = 2$) the oscillations of $\tilde{\rho} \equiv a\rho$ are given by an elliptic cosine and Eq. (8) reads as an equation in Minkowski space-time [7] (technically speaking it is a Lame equation). The resonance structure and the relative Floquet exponents μ_k depend non-monotonically by the ratio e^2/λ_2 [7]: long wavelengths $k^2 \ll \lambda_2 \tilde{\rho}_0^2$ are resonant for $1/2\pi < e^2/\lambda_2 < 3/2\pi$, as well for other values [7], where $\tilde{\rho}_0$ is the initial amplitude for $\tilde{\rho}$. For $n \geq 3$ the driving term $a^2\rho^2$ oscillates and *grows* in time as $\eta^{\frac{2(n-2)}{2n-1}}$. In this case the resonance for A_{Tk} (as well for $a\chi$) is very efficient [15].

The case of a simmetry breaking potential

$$V = m^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 \tag{12}$$

with $m^2 < 0$, is definitively the closest to the electroweak phase transition: Φ would play the role of the Higgs field, and ρ rolls down from the zero value (false vacuum) towards $\rho_{min} = \pm\sqrt{m^2/\lambda}$ (true vacuum). Let us note that the most important feature in the case of symmetry breaking in this toy model of scalar electrodynamics is the Higgs mechanism: the gauge field gets an effective mass proportional to the value of the field in the broken phase $\sqrt{m^2/\lambda}$. Besides the unpleasant feature of considering a massive photon, this fact could completely inhibit the resonance in an expanding universe or narrow the resonance shifting it to scales proportional to $\sqrt{m^2/\lambda}$. However, in the realistic electroweak theory the photon does not acquire a mass: the symmetry breaking mechanism gives masses to the W and Z bosons, leaving the photon massless. This is an important condition for the efficiency of parametric resonance in expanding universe. We note that in this case the driving term in Eq. (8) is given by the W condensate [16].

Let us discuss for completeness also the behaviour of the scalar field fluctuations for the potential (12) without any constraints on the sign of m^2. The Fourier components of the real part of the rescaled field fluctuation $\widetilde{\delta\phi_1} \equiv a\delta\phi_1$ satisfy the following equation:

$$\widetilde{\delta\phi}''_{1\,k} + \left[\omega^2_{1\,k} - \frac{a''}{a}\right] \widetilde{\delta\phi}_{1\,k} = 0 \tag{13}$$

where $\omega^2_{1\,k} = k^2 + a^2 m^2 + 3\lambda a^2 \rho^2$. The Fourier components of the imaginary part of the rescaled field fluctuation $\widetilde{\delta\phi_2} \equiv a\delta\phi_2$ satisfy

$$\widetilde{\delta\phi}''_{2\,k} + \left[\omega^2_{2\,k} - \frac{a''}{a}\right] \widetilde{\delta\phi}_{2\,k} = ea^2(2\delta A_{0\,k}\rho' + \rho\delta A'_{0\,k} + 3aH\rho\delta A_{0\,k}) \tag{14}$$

where $\omega^2_{2\,k} = k^2 + m^2 a^2 + \lambda a^2 \rho^2$. By using the constraint equation (7) and introducing $q_k = \widetilde{\delta\phi}_{2\,k}/\omega_T$ one obtains

$$q''_k + \left[\omega^2_{k\,2} + \omega^2_T - k^2 - \frac{a''}{a} + \frac{8\pi e^2 a^2}{\omega^2_T}(\rho' + aH\rho)^2 + \frac{\omega''_T}{\omega_T} - 2\frac{\omega'^2_T}{\omega^2_T}\right] q_k = 0\,. \tag{15}$$

We note that the gauge coupling affects the resonance structure of the scalar field: while ω_{1k}^2 is the driving term for $\widetilde{\delta\phi}_{1k}$, ω_{2k}^2 does not determine the resonance bands for $\widetilde{\delta\phi}_{2k}$, as would do if the charged scalar field were uncoupled to a gauge field [17].

Besides the PR effect, one of the two circular polarization exhibit also a negative effective mass (depending on the sign of $\dot{\phi}$). This feature could be interesting before the stage during which the field coherently oscillates, when $\dot{\phi}$ has a fixed sign.

III. AXION ELECTRODYNAMICS

We consider the effect of parametric resonance for another type of coupling described by:

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{g}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{16}$$

The neutral scalar field ϕ can represent an axion in [18] (in such a case the vacuum angle θ is defined to be $\theta = \phi/f$, where f is the Peccei-Quinn simmetry scale) or a general pseudo-Goldstone boson in [19]. By considering again the scalar field as $\phi(t, \mathbf{x}) = \phi(t) + \delta\phi(t, \mathbf{x})$ one has at the homogeneous level:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \tag{17}$$

and for the transverse degrees of freedom

$$\delta\ddot{\mathbf{A}}_T + H\delta\dot{\mathbf{A}}_T - \frac{\nabla^2}{a^2}\delta\mathbf{A}_T + 4\pi g\dot{\phi}\frac{\nabla}{a} \times \delta\mathbf{A}_T = 0\,. \tag{18}$$

In this case it is more convenient to consider the circular polarized Fourier component perpendicular to $\mathbf{k}$, $\mathbf{A}_{\pm\,k}$, whose equation of motion is

$$\delta\mathbf{A}''_{\pm\,k} + (k^2 \pm 4\pi g\phi' k)\delta\mathbf{A}_{\pm\,k} = 0 \tag{19}$$

In the regime of coherent oscillation of the scalar field one has again a Mathieu-like equation where the driving term is proportional to the time derivative of the scalar field and goes as η^{-1} for a power law potential analogous to Eq. (11) (always under the assumption that the scalar field dominates the energy density of the universe [14]). The efficiency of the resonance depends on the value of the adimensional quantity $g\phi$. The analysis of Eq. (19), describing the axion decay into photons, was probably one of the first application of parametric resonance in cosmology [20], and it has been recently reanalyzed in [21,22]. By considering ϕ as the QCD axion, the smallness of $gf \sim 10^{-4}$ prevents the efficiency of the resonance in the case of a potential with temperature-dependent mass due to QCD effects [21]. In general the resonance is on $k \sim 4\pi g\phi\omega$, where ω is the oscillation frequency of ϕ. However, for $V = \lambda\phi^4/4$ and $4\pi gf = 1$ we observe a linear growth of $A_\pm$ for $k/\omega \ll 4\pi gf$ by numerically integrating Eq. (17) and (19). Such growth is slower than the quasi-exponential amplification of the field fluctuations $\delta\phi$ on smaller scales [7].

IV. PLASMA EFFECTS

We now comment on effects due to the presence of other charged particles *. These effects can be important since the universe is considered a good conductor after reheating until decoupling. For wavelengths larger than the collision length [23] one could use the Ohm relation for the additional current $\mathbf{j}_{\rm add} = \sigma\mathbf{E}$. This would add to Eqs. (8) and (18) a damping term:

$$\delta\mathbf{A}''_k + 4\pi(a\sigma)\delta\mathbf{A}'_k + \omega_k^2(\eta)\delta\mathbf{A}_k = 0\,, \tag{20}$$

where we have denoted collectively the transverse component as $\delta\mathbf{A}$ and with ω_k^2 their time dependent frequency. The time behaviour of conductivity is considered in the literature to be $\sigma \propto T \propto 1/a$ [18] and so the term multiplying the first derivative in Eq. (20) is constant: in this way the resonance on long wavelengths could be washed out if $a\sigma$ is larger than the Floquet exponent. For wavelengths smaller than the collision length charge separation should be taken into account and a term qualitatively similar to the classical plasma frequency would enter as:

$$\omega_k^2(\eta) \to \omega_k^2(\eta) + 4\pi e^2 \frac{n(\eta)}{m} \tag{21}$$

where $n(\eta)$ and m are respectively the number density and the mass of the charged particles. This term would play the role of an effective mass, but which would decay as $a(t)^{-3}$ (which is more rapid than the decay of the driving term considered in this paper), leaving open the possibility of resonance, in particular for large coupling costants. Let us observe that these qualitative estimates for conductivity and plasma frequency are obtained by thermodynamic considerations. Even in the case of a preexisting thermal equilibrium the resonance would amplify the gauge fluctuations. Infact a thermal mass $\sim T^2$ for the gauge fiels will not be able to prevent the resonance and it has the effect of shifting the resonance on smaller scales (k^2 must be replaced with $k^2 + g_{\rm eff} T^2 a^2$).

There is an epoch where the the resonance could proceed just as worked out in the vacuum case: in preheating after inflation, when it is assumed that the "creation"

*We are referring to charged fields which are uncoupled from the scalar field which oscillates coherently. These fields produce currents J_μ which are *incoherent* and are coupled in a $J_\mu A^\mu$ to the gauge field.

of charged particles is contemporaneous to the amplification of gauge fluctuations. This amplification will last until rescattering and backreaction set in to shut off the resonance [24], and terms such as conductivity and plasma frequency will appear in this out-of-equilibrium process by a self-consistent study of the problem [25](although their time-dependence will be different from the corresponding thermal equilibrium quantities). In this case the constraints on the coupling costants e^2 and g come only from requirement to protect the potential of the scalar field from radiative corrections: this argument would give the usual $e^2 \lesssim 10^{-6}$ for the scalar electrodynamic case. If one requires that the charged inflaton gives the right amount of e-folds and CMB fluctuations ($m \sim 10^{-6} M_{pl}$ for a massive inflaton, $\lambda \sim 10^{-13}$ for a self-interacting inflaton), then a broad resonance regime would be allowed for the gauge field. For the axion electrodynamic case, the same argument leads to $gf \gg 10^{-4}$, opening a possible new efficient channel for the decay of the scalar field ϕ into gauge fluctuations.

V. PRIMORDIAL MAGNETIC FIELD BY PARAMETRIC RESONANCE?

Even if the conformal property of gauge fields in cosmological backgrounds is useful for PR, it is the main reason why gauge fluctuations are not amplifield by the expansion of the universe [9], in contrast to what happens for minimally coupled scalar fields. Therefor the conformal invariance of gauge fields is the main issue for the explanation of primordial magnetic fields [26] within the inflationary paradigm [18,19].

PR could be interesting for the generation of primordial magnetic fields for two reasons. First, it is intringuing to note how the exponential growth due to parametric resonance is of the same type as the dynamo effect [26], one of the astrophysical processes postulated to explain the observational evidence for galactic magnetic fields. As a second point, there are several examples in which significant amplification occurs on the maximum causal scale allowed by the problem (the coherence scale of the field). Since the scale of the magnetic seeds is always a crucial issue for a model which aims to explain their origin, this feature of PR is very interesting.

As a straightforward result of section II preheating could provide an extra growth in the amplitude of gauge fluctuations which are produced during inflation [18,19]: for parameters in which long wavelength gauge fluctuations are amplified, this occurs directly on observable scales because of the coherence of the inflaton on the particle horizon scale.

We illustrate how the mechanism could work in a simple scalar electrodynamic toy model where the charged scalar field is a massless self-interacting inflaton with $2\pi e^2 = \lambda$ During inflation, assuming Φ in slow rollover, a solution for Eq. (8) is:

$$\delta \mathbf{A}_{T\,k} = (-H\eta)^{1/2} \left(\frac{\pi}{4H}\right)^{1/2} H_\nu^{(1)}(-k\eta) \qquad (22)$$

where $\eta = -1/(Ha)$ is the conformal time used to model a de Sitter era ($-\infty < \eta < \eta_0 < 0$, where η_0 is some arbitrary time) and the index of the Hankel function is so defined

$$\nu^2 = \frac{1}{4} - 4\pi e^2 \frac{\rho^2}{H^2} \simeq \frac{1}{4} - \frac{3e^2}{2\lambda} \frac{M_{\rm pl}^2}{\rho^2} \qquad (23)$$

where the last equality holds during slow-rollover and $M_{\rm pl}^2$ is the inverse of the Newton constant G. The solution (22) matches the adiabatic vacuum $e^{-ik\eta}/\sqrt{2k}$ for $\eta \to -\infty$. For $\nu^2 > 0$ Eq. (22) leads to a slight shift of the initial vacuum for the long wavelength fluctuations ($-k\eta \to 0$) of the gauge field:

$$\delta \mathbf{A}_{T\,k} \simeq -i \frac{\Gamma(\nu)}{\sqrt{2\pi k}} \left(\frac{-k\eta}{2}\right)^{\frac{1}{2}-\nu} . \qquad (24)$$

By using the definition $B_i = \epsilon_{imn} \partial_m A_n / a$ one can relate the fluctuation of the magnetic field to the gauge field fluctuations

$$|\mathbf{B}_k|^2 = \frac{k^2 |\delta \mathbf{A}_{T\,k}|^2}{a^4} \qquad (25)$$

Parametric resonance after inflation amplifies the gauge fluctuations in Eq. (24) and $|\mathbf{B}_k|^2$ in Eq. (25) exponentially up to a maximum factor 10^{12} [24].

However, this last amplification is not sufficient to explain the value required by the observation. Indeed, the simple toy model of a self-interacting inflaton is very useful in order to follow gauge fluctuations at a linear order from inflation through preheating, but it misses a super-adiabatic amplification during inflation, which was investigated by Turner and Widrow [18]. This amplification can occur due to an effective *negative* mass for the gauge field during inflation, which can be obtained by breaking the conformal invariance through a explicit coupling to the curvature, for instance [18]. In string cosmology an effective negative mass for the gauge field is generated by the dilaton coupling [27].

Instead, as we see from Eq. (24), long wavelength gauge fluctuations are slightly modified in amplitude and spectrum with respect to the initial adiabatic spectrum by the coupling with inflaton during slow-rollover. Indeed the *positive* mass generated by the coupling of the gauge field to the charged scalar field lead to a slight suppression. When slow-rollover is ended and $\nu^2 < 0$ the gauge modes are suppressed. This phase is just a short transient, after which the resonant amplification starts immediately (roughly when ρ first cross zero). The main point is the decay of B^2 as $1/a^4$ once a gauge fluctuation has left the Hubble radius during inflation, leading to a suppression which the following preheating phase cannot fill up [28].

Now we quantify our previous statements and we estimate the order of magnitude of the amplitude for the magnetic field at decoupling. We shall consider the effect of preheating as a k-independent amplification factor Ω^2 which multiplies the spectrum at the end of inflation (24). We also neglect the finite time duration of the preheating phase and we identify the end of inflation with the beginning of the usual radiation dominated Friedmann era. At the beginning of the radiation dominated phase (denoted in the following by η^*) the spectrum of the magnetic field is:

$$k^3 |\mathbf{B}_k(\eta^*)|^2 = \Omega^2 \frac{k^5}{a^4(\eta^*)} |\delta \mathbf{A}_{T\,k}(\eta^*)|^2 \tag{26}$$

where we consider $\delta \mathbf{A}_{T\,k}(\eta^*) \simeq 1/\sqrt{2k}$, which has been obtained from Eq. (24) by setting $\nu = 1/2$. By using the the relation $a^4 B^2 = const.$ for the magnetic field evolution in the radiation dominated era, the spectrum at the decoupling time $\eta_{\rm DEC}$ is:

$$k^3 |\mathbf{B}_k(\eta_{\rm DEC})|^2 = k^3 |\mathbf{B}_k(\eta^*)|^2 \frac{T^4_{\rm DEC}}{T^4_{\rm REH}} \tag{27}$$

since the temperature T scales as the inverse of the scale factor a. By using Eqs. (26) and (27) we estimate at time of decoupling a magnetic field of size

$$\begin{aligned} \mathcal{B}|_{\rm DEC}(k) &= (k^3 |\mathbf{B}_k(\eta_{\rm DEC})|^2)^{1/2} \\ &\simeq \frac{\Omega}{\sqrt{2}} (k{\rm kpc})^2 \frac{T^2_{\rm DEC}}{{\rm kpc}^2 T^2_{\rm NOW}} \\ &\simeq 10^{-42}\,{\rm Gauss} \end{aligned} \tag{28}$$

where it has been assumed $a(\eta_{\rm NOW}) = 1$, a comoving wavenumber $k = 0.1\,{\rm kpc}^{-1} = 0.64 \times 10^{-36}{\rm GeV}$, $\Omega = 10^6$, $T_{\rm DEC}/T_{\rm NOW} \simeq 10^3$, $1\,{\rm Gauss} = 3 \times 10^{-20}{\rm GeV}^2$ [29]. The estimate (28) is lower than the magnetic seed field required by observation, which is 10^{-34}G for a flat, low density universe [30]. We note that $\mathcal{B}$ is proportional to Ω, but not to $T_{\rm REH}$ or to the Hubble constant during inflation, as observed in [18]. This is due to the the approximation $\nu = 1/2$ which restablishes the conformal invariant spectrum in the estimate (28).

VI. CONCLUSIONS

In this paper we have studied the possibility of an enhancement of gauge fields due to parametric resonance. For both the couplings to a scalar charged field and to an axion, the resonance can counteract the redshift due to the expansion of the universe and exponential growing modes appear. The resonance is generically on small scales in the axion case, while it can occur for scales up to the coherence scale of the scalar field in the scalar electrodynamic case. Therefore the latter scalar field coupling is more interesting for the problem of cosmological magnetic fields, in which the large scale observed seems a crucial issue.

In the simplest toy model with a massless charged inflaton, a linear calculation based on preheating does not provide directly an amplitude in agreement with observations because the magnetic field fluctuations are redshifted during inflation. However, this latter effect may be circumvented in many ways. In more realistic inflationary models motivated by particle physics the gauge field can be sustained during inflation and then amplified during preheating. A sustain during inflation could be also realized by breaking conformal invariance with explicit coupling between the gauge field and the Riemann curvature in the lagrangian [18]. Conformal invariance is also effectively broken by the inhomogeneities genarated during inflation (a second order effect in perturbations). Moreover, PR would also provide an additional growth in the generation mechanism based on the statistical correlation of the currents studied in [11].

A final amplitude and spectrum of gauge field fluctuations generated by PR is however beyond the scope of the present paper. Infact, its calculation would include the analysis of nonlinear effects and a simple estimate based on matching techniques is inadequate. In models with interacting scalar fields, nonlinear effects as rescattering [24] play an important role in the generation of the final power spectra. Indeed, such power spectra, as $|\delta \mathbf{A}_k|^2 \sim k^{-\alpha}$ with $\alpha > 1$, would be very interesting for the problem of large scale magnetic fields, because steeper than (24) in the infrared region.

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